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LETTERS TO THE EDITOR

TRANSVERSE VIBRATIONS OF A CIRCULAR, ANNULAR PLATE WITH FREE EDGES AND TWO, INTERMEDIATE CONCENTRIC CIRCULAR SUPPORTS

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1. INTRODUCTION

The present note deals with the determination of the fundamental frequency coefficient of transverse vibration of the structural system depicted in Figure 1 and which, apparently, has not been previously considered by other researchers. Two independent solutions are obtained in the present study using: (1) the optimized Rayleigh–Ritz method [1], (2) a finite element algorithmic procedure [2].

2. APPROXIMATE, ANALYTICAL SOLUTION

In the case of axisymmetric normal modes of vibration the governing function is

$$J(W) = D\int_b^a \left[\left(\mathrm{W}^n + \frac{W'}{\bar{r}}\right)^2 + 2(1-v)\frac{W'W''}{\bar{r}}\right]\bar{r}\,\mathrm{d}\bar{r} - \rho h\omega^2\int_a^b W^2\bar{r}\,\mathrm{d}\bar{r}, \qquad (1)$$

where $W(r)$ is the normal-mode amplitude.

Introducing the dimensionless variable $r = \bar{r}/a$ and substituting in equation (1) results in

$$\frac{a^2}{D}J(W) = \int_{r_b}^1 \left[\left(W'' + \frac{W'}{\bar{r}}\right)^2 - 2(1-v)\frac{W'W''}{\bar{r}}\right]\bar{r}\,\mathrm{d}r - \Omega^2\int_{r_b}^l W^2 r\,\mathrm{d}r, \qquad (2)$$

where

$$\Omega^2 = \frac{\rho h a^4}{D}\omega^2, \qquad r_b = \frac{b}{a}, \qquad r_c = \frac{c}{a}, \qquad r_d = \frac{d}{a}.$$

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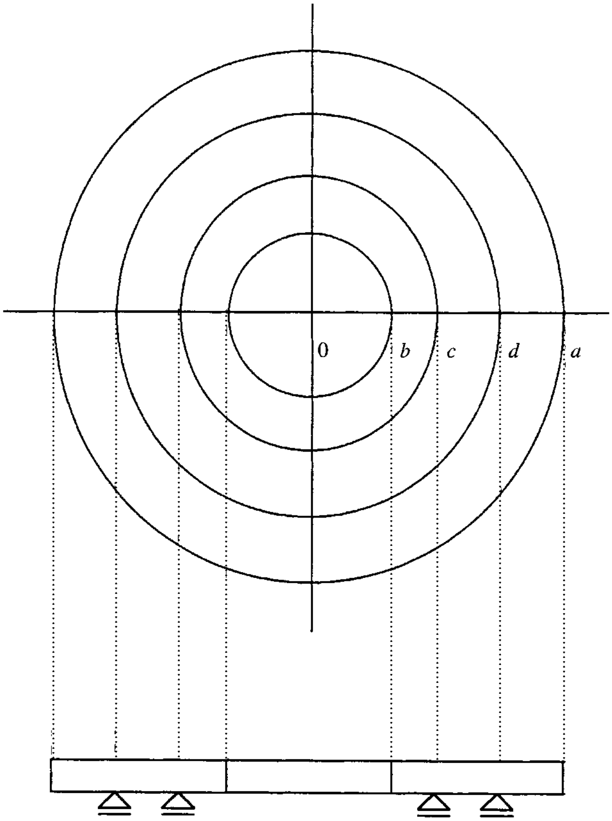

Figure 1. Vibrating mechanical system under study.

The present investigation makes use of polynomial co-ordinate functions which satisfy the natural and essential boundary conditions

$$W''(r_b) + \frac{\nu}{r_b} W'(r_b) = 0, \qquad W(r_c) = W(r_d) = 0, \qquad W''(1) + \nu W'(1) = 0. \quad (3)$$

Accordingly and following previous studies [3, 4] the approximation

$$W_a = \sum_{j=1}^{N} C_j \varphi_j(r) = \sum_{j=1}^{N} C_j (a_j r^{p+j-1} + b_j r^{j+2} + c_j r^{j+1} + d_j r^j + 1), \quad (4)$$

is used where the a_j's, b_j's and d_j's are determined substituting each polynomial co-ordinate function in equation (3).

On the other hand, the approximating function does not satisfy the condition of nulle shear force at the free edges of the plate. This is certainly a legitimate procedure since the Rayleigh–Ritz method is employed, on the one hand, and on the other, it turns out that the methodology is applicable when the edges $r = a$, b are simple supported.

Substituting equation (4) in equation (2) and making use of the well-established Rayleigh–Ritz procedures leads to a secular determinant in the eigenvalues of the

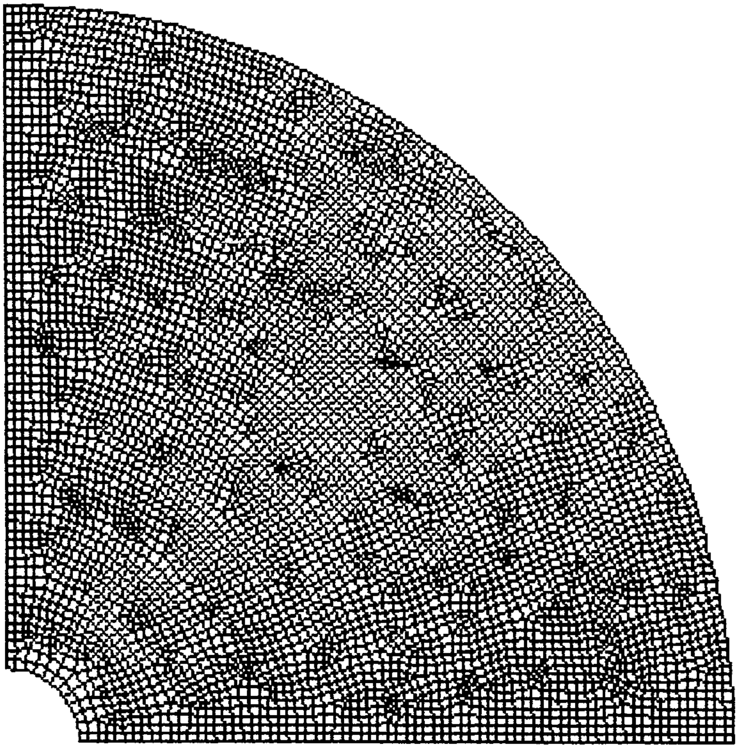

Figure 2. Finite element mesh ($r_b = 0{\cdot}1$).

problem. Its lowest root constitutes the fundamental frequency coefficient $\Omega_1 = \sqrt{(\rho h/D)}\omega_1 a^2$.

Since

$$\Omega_1 = \Omega_1(p), \tag{5}$$

by minimization of Ω_1 with respect to p, one obtains an optimized value of the fundamental frequency coefficient.

3. FINITE ELEMENT SOLUTION

Finite element determinations were carried out using ALGOR [2]. Because of symmetry considerations one-quarter of the plate* was subdivided into 5835 elements. Accordingly, 5981 nodes resulted and taking into account boundary and double symmetry conditions, a mathematical model of 17 728 degrees of freedom was generated; see Figure 2.

4. NUMERICAL RESULTS

All calculations were performed taking the Poisson ratio ν equal to 0·3. On the other hand, the analytical determinations were carried out making $N = 10$ in

*For $r_b = 0{\cdot}1$. For other values of r_b considerably less elements were taken, see Table 2.

TABLE 1

Values of Ω_1 of the system shown in Figure 1: values obtained by means of the optimized Rayleigh–Ritz method

r_b	r_c	$r_d = 0{\cdot}3$	$r_d = 0{\cdot}4$	$r_d = 0{\cdot}5$	$r_d = 0{\cdot}6$	$r_d = 0{\cdot}7$	$r_d = 0{\cdot}8$
0·1	0·2	6·14	7·56	9·75	13·37	19·59	26·28
	0·3		8·21	10·54	14·47	21·56	32·02
	0·4			11·64	15·84	23·70	36·38
	0·5				17·86	26·09	31·41
	0·6					25·57	24·06
	0·7						19·22
0·2	0·3		8·19	10·50	14·38	21·35	31·46
	0·4			11·62	15·81	23·66	37·50
	0·5				17·85	26·29	36·45
	0·6					28·34	27·08
	0·7						20·88
0·3	0·4			11·57	15·78	23·61	38·19
	0·5				17·77	26·47	43·73
	0·6					30·36	36·65
	0·7						26·27
0·4	0·5				17·73	26·53	45·54
	0·6					30·59	51·21
	0·7						39·22
0·5	0·6					30·60	53·75
	0·7						62·43
0·6	0·7						64·86

TABLE 2

Number of elements and nodes for the structural configurations considered when using the finite element method

r_b	No. elements	No. nodes
0·1	5835	5981
0·2	5668	5813
0·3	5383	5527
0·4	4952	5095
0·5	4595	4454
0·6	3815	3955

equation (4). Table 1 depicts values of Ω_1 calculated by means of the optimized Rayleigh–Ritz method for several combinations of values of the geometric parameters r_b, r_c and r_d.

Table 3 contains values of Ω_1, determined by means of the finite element algorithmic procedure. By comparing values of the fundamental frequency

TABLE 3

Values of Ω_1 determined by means of the finite element method

r_b	r_c	$r_d = 0{\cdot}3$	$r_d = 0{\cdot}4$	$r_d = 0{\cdot}5$	$r_d = 0{\cdot}6$	$r_d = 0{\cdot}7$	$r_d = 0{\cdot}8$
0·1	0·2	6·134	7·550	9·744	13·362	19·581	26·277
	0·3	—	8·174	10·530	14·456	21·542	31·997
	0·4	—	—	11·563	15·821	23·663	36·360
	0·5	—	—	—	17·692	26·056	31·395
	0·6	—	—	—	—	25·517	24·038
	0·7	—	—	—	—	—	19·181
0.2	0·3	—	8·165	10·494	14·364	21·331	31·440
	0·4	—	—	11·555	15·791	23·624	37·471
	0·5	—	—	—	17·691	26·251	36·415
	0·6	—	—	—	—	28·320	27·045
	0·7	—	—	—	—	—	20.788
0·3	0·4	—	—	11·548	15·763	23·591	38·146
	0·5	—	—	—	17·693	26·431	43·675
	0·6	—	—	—	—	30·225	36·605
	0·7	—	—	—	—	—	26·188
0·4	0·5	—	—	—	17·692	26·489	45·485
	0·6	—	—	—	—	30·472	51·134
	0·7	—	—	—	—	—	39·113
0·5	0·6	—	—	—	—	30·529	53·641
	0·7	—	—	—	—	—	62·296
0·6	0·7	—	—	—	—	—	64·649

coefficients obtained by both independent approaches (Tables 1 and 3) one concludes that the agreement is excellent for all the situations considered.

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